

Multichannel chiral approach for kaonic hydrogen

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Abstract

We present an exact solution to the K^- -proton bound state problem formulated in the momentum space. The 1s level characteristics of the kaonic hydrogen are described together with the available low energy $\bar{K}N$ data.

1 Introduction

We developed a precise method of computing the meson-nuclear bound states in momentum space. The method was already applied to pionic atoms and its multichannel version was used to calculate the 1s level characteristics of pionic hydrogen [1]. Here we just remark that our approach is based on the construction of the Jost matrix and involves the solution of the Lippman-Schwinger equation for the transition amplitudes between the various channels. In this work we aim at simultaneous description of both the 1s level kaonic bound state and the available experimental data for the K^-p initiated processes.

In view of the vastly improved experimental results on the 1s level of kaonic hydrogen [2] the exact solution of the bound state problem should be preferred over the traditional Deser-Trueman formula relating the threshold scattering amplitude to the hadronic energy level characteristics of exotic atoms. Recently, the relation for kaonic hydrogen was modified to include the isospin effects and electromagnetic corrections [3].

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2 Meson-baryon potentials

Unlike the pion-nucleon interaction the $\overline{K}N$ dynamics is strongly influenced by the existence of the $\Lambda(1405)$ resonance, just below the K^-p threshold. This means that the standard chiral perturbation theory is not applicable in this region. Fortunately, one can use non-perturbative coupled channel techniques to deal with the problem and generate the $\Lambda(1405)$ resonance dynamically. Such approach has proven quite useful and several authors have already applied it to various low energy meson-baryon processes [4–6].

Here we follow the approach of Ref. [4] and take the strong interaction part of the potential matrix in a separable form

$$V_{ij}(k, k') = \sqrt{\frac{1}{2E_i} \frac{M_i}{\omega_i}} g_i(k) \frac{C_{ij}}{f^2} g_j(k') \sqrt{\frac{1}{2E_j} \frac{M_j}{\omega_j}}, \quad g_j(k) = \frac{1}{1 + (k/\alpha_j)^2} \quad (1)$$

in which the parameter f stands for the pseudoscalar meson decay constant in the chiral limit. The coupling matrix C_{ij} is determined by chiral SU(3) symmetry and it includes terms up to second order in the meson c.m. kinetic energies. The off shell form factor $g_j(k)$ introduces the inverse range radius α_j that characterizes the radius of interaction in the channel j . In the Born approximation the potentials $V_{ij}(k, k')$ give the same (up to $\mathcal{O}(q^2)$) s-wave scattering lengths as are those derived from the underlying chiral lagrangian. More details on the construction of the effective (chirally motivated) potentials and the specification of the kinematical factors $\sqrt{M_j/(2E_j\omega_j)}$ can be found in Refs. [4, 7]. While the authors of Ref. [4] restricted themselves only to the six channels that are open at the $\overline{K}N$ threshold we have employed all ten coupled meson-baryon channels in our model: K^-p , \overline{K}^0n , $\pi^0\Lambda$, $\pi^+\Sigma^-$, $\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\eta\Lambda$, $\eta\Sigma^0$, $K^+\Xi^-$, and $K^0\Xi^0$. The potential of Eq. (1) is used not only when solving the bound state problem but we also implement it in the standard Lippman-Schwinger equation and compute the low energy $\overline{K}N$ cross sections and branching ratios from the resulting transition amplitudes.

3 $\overline{K}N$ data fits

The parameters of the chiral lagrangian that enter the coefficients C_{ij} and the inverse range radii α_i determining the off-shell behavior of the potentials are to be fitted to the experimental data. Before performing the fits we reduce the number of the fitted parameters in the following way. First, the axial couplings D and F (concerning the specification of the various chiral couplings we refer the reader to Refs. [4] and [7]) have already been established in the analysis of semileptonic hyperon decays, $D = 0.80$, $F = 0.46$

($g_A = F + D = 1.26$). Then, we fix the couplings b_0 and b_F to satisfy the approximate Gell-Mann formulas for the baryon mass splittings, $b_0 = 0.064 \text{ GeV}^{-1}$ and $b_F = -0.209 \text{ GeV}^{-1}$. Similarly, we determine the coupling b_0 and the baryon chiral mass M_0 from the relations for the pion-nucleon sigma term $\sigma_{\pi N}$ and for the proton mass (see e.g. [6]). Finally, we reduce the number of the inverse ranges α_i to only five: α_{KN} , $\alpha_{\pi\Lambda}$, $\alpha_{\pi\Sigma}$, $\alpha_{\eta\Lambda/\Sigma}$, $\alpha_{K\Xi}$. This leaves us with 11 free parameters: the five inverse ranges, the pseudoscalar meson decay constant f , and five more couplings from the second order chiral lagrangian denoted by d_D , d_F , d_0 , d_1 , and d_2 .

The fits to low energy $\overline{K}N$ data standardly include the three precisely measured threshold branching ratios γ , R_c and R_n (specified e.g. in Ref. [4]) and the K^-p -initiated total cross sections. For the later ones we consider only the experimental data taken at the kaon laboratory momenta $p_{LAB} = 110 \text{ MeV}$ (for the K^-p , \overline{K}^0n , $\pi^+\Sigma^-$, $\pi^-\Sigma^+$ final states) and at $p_{LAB} = 200 \text{ MeV}$ (for the same four channels plus $\pi^0\Lambda$ and $\pi^0\Sigma^0$). Our results show that the inclusion of more data taken at other kaon momenta is not necessary since the fit at just 1 – 2 points fixes the cross section magnitude and the energy dependence is reproduced nicely by the model. With the inclusion of the DEAR results on the strong interaction shift ΔE_N and the width Γ of the 1s level in kaonic hydrogen we end up with a total of 15 data points in our fits.

Table 1: The fitted $\overline{K}N$ threshold data

$\sigma_{\pi N} [\text{MeV}]$	χ^2/N	$\Delta E_N [\text{eV}]$	$\Gamma [\text{eV}]$	γ	R_c	R_n
20	1.33	232	725	2.366	0.657	0.191
30	1.36	272	683	2.367	0.658	0.190
40	1.38	257	713	2.370	0.658	0.190
50	1.40	266	708	2.370	0.658	0.190
exp	-	193(43)	249(150)	2.36(4)	0.664(11)	0.189(15)

Our results are summarized in Table 1, where the results of our χ^2 fits are compared with the relevant experimental data. Since the value of the pion-nucleon σ -term is not well determined we enforced four different options, which cover the interval of the values considered by various authors. The resulting χ^2 per data point indicate satisfactory fits. It is worth noting that their quality and the computed values do not depend much on the exact value of the $\sigma_{\pi N}$. The low energy cross sections included in the fits are not shown here but we stress that their description is good [7]. The strong interaction energy shift of the 1s level in kaonic hydrogen is reproduced well but we were not able to get a satisfactory fit of the 1s level energy width as our results are significantly larger than the experimental value. However, when considering the interval of three standard deviations and also the older KEK results (that

give less precise but larger width) one cannot conclude that kaonic hydrogen measurements contradict the other low energy $\overline{K}N$ data.

In Table 2 we compare our results (for $\sigma_{\pi N} = 20, 30$ and 50 MeV) for the 1s level characteristics in kaonic hydrogen with the approximate values determined from the K^-p scattering lengths a_{K^-p} that were obtained from the multiple channel calculation that uses the same parametrization of the strong interaction potential (1). The 1s level complex energies are shown for: the standard Deser-Trueman formula (DT), the modified Deser-Trueman formula (MDT) [3] and our “exact” solution of the bound state problem. We have checked that if only the point-like Coulomb potential is considered in the K^-p channel our method reproduces the well known Bohr energy of the 1s level with a precision better than 0.1 eV. This means that the discrepancy between the “MDT” and the “exact” values can be attributed to higher order corrections not considered in the derivation of the MDT. In view of the current level of the experimental precision the use of the MDT formula is sufficient. Though, the situation may change after the coming SIDDHARTA experiment.

Table 2: Precision of the Deser-Trueman formula

a_{K^-p} [fm]	$-0.50 + i 1.01$	$-0.59 + i 0.99$	$-0.60 + i 1.01$
DT: $\Delta E_N - (i/2)\Gamma$ [eV]	$207 - (i/2)832$	$256 - (i/2)806$	$247 - (i/2)830$
MDT: $\Delta E_N - (i/2)\Gamma$ [eV]	$251 - (i/2)714$	$290 - (i/2)664$	$285 - (i/2)689$
exact: $\Delta E_N - (i/2)\Gamma$ [eV]	$232 - (i/2)725$	$262 - (i/2)698$	$266 - (i/2)708$

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